The Equation of Life

The equation links five quantities to the Pupil Teacher Ratio a school can afford in a balanced budget set for an academic year. The terms are defined below and the derivation of the equation is also given.

The equation is

$$PTR = \frac{ATC}{I \times p_T} = c \times ACS$$

Where

PTR is Pupil to teacher ratio

ATC is Average Teacher Cost

I is revenue available per pupil

P_T is proportion of revenue available for expenditure on teacher cost

c is teacher contact ratio

ACS is average class size

There are other ways of calculating the number of teachers a school can afford to employ given estimates of funding, teaching salaries plus on cost and all other school expenditure but I have found the format above to make the calculation in terms of PTR is quite useful in a number of ways. Firstly, the quantities in the equation lend themselves easily to benchmarking activity. Secondly the link to the curriculum plan and hence the timetable through c and ACS is a simple one.

A third way of thinking of these terms is to estimate them for a given year and then view them as targets. Unexpected events such as long term illness or planned situations such as introducing increased management time change these targets and imply adjustment in other values in order to maintain a balance. The equation also provides a mechanism for looking at the finances from a non-financial point of view which also has the advantage of being in terms of only a few quantities.

Given a value for the PTR that the school can afford and a typical value for the teacher contact ratio the value of ACS can be calculated. Understanding the meaning of this number in the context of a school is a little like a doctor understanding the possible implications of a blood pressure value.

A value of ACS implies a budget of teacher periods for the curriculum plan which can be planned in the context of a staff deployment analysis.

Technical terms and basic relationships

Note that the relationships below apply to an academic year. If a school is working in financial years the values of ACS, I and p_T should be estimated for the academic year that the teacher contact ratio, Pupil to Teacher Ratio and Average Class Size relate to

PTR

This is the pupil to teacher ratio found by dividing the pupil roll by the total number of teachers expressed in terms of Full Time Equivalent (FTE)

ATC

This the Average Teacher Cost and is the total salary plus on costs of all the teachers counted in PTR calculation divided by the FTE teaching staff total. It is probably useful to include supply and agency costs in addition to salary and on costs.

I

(The symbol is a capital letter I). This is the revenue funding in pounds per pupil. This is the sum of all funding, income and anything else which can be legitimately counted in the global pot of money to spend on revenue items divided by the number of pupils. I suggest that any carry forward is excluded so the in-year position can be assessed. If a school is working in financial years the value of *I* used should be estimated for the academic year in question.

p_T

This is the proportion of revenue available for spending on teachers. It is the decimal fraction produced when the total revenue budget is divided into the sum of all expenditure lines not included in the ATC as defined above.

c

This is the teacher contact ratio. It is the average teaching load of the FTE teacher total used in the PTR and ATC calculations expressed as a fraction of the timetable cycle. It can be calculated by taking the total number of available teaching periods and dividing that by the product of the periods in the timetable cycle and the FTE teacher total used in earlier sections. It is typically a decimal in the 0.6 to 0.8 area. Values as high as 0.8 are now seem extremely rare. 0.8 was the norm around 30 years ago but school organisation now has quite different demands on it. I have only seen values for *c* as low as 0.6 in situations where schools are generously funded. More frequently, the values I have observed are in the mid-range between 0.7 and 0.8. For the last five years I have advocated 0.78 as an aspirational target on the basis that that represents approximately 10 per cent of all teacher time in planning and preparation, 10 per cent in management activity and allows 2 per cent error since 10+10+2 = 22 and 100-22 =78.

ACS

This is the average class size. This is a global statistic for an institution and does not actually represent any particular class. It is the number of pupils that would be allocated to a teacher if all the teachers who are teaching on any period of the week, on average shared all the pupils on roll evenly between them. There are two relationships for calculating it

The first follows from the definition.

If *tp* represents the number of teacher contact periods on the whole timetable and *w* represents the number of periods there are in the timetable cycle then the average number of teachers in contact with students on any one period is given by

$$\frac{tp}{w}$$

If the number of pupils on roll, *N*, is divided by this fraction we get the average class size so (N.B I have inserted multiplication signs to distinguish between separate symbols and symbols that have more than one letter.

$$ACS = \frac{N \times w}{tp}$$

The second is

$$ACS = \frac{PTR}{c}$$

This follows by substitution since

$$c = \frac{tp}{T \times w}$$

where *T* is the full time equivalent number of teachers.

And

$$PTR = \frac{N}{T}$$

The second relationship for ACS is **one of the two basic relationships** needed to relate finance and curriculum (As above I have inserted multiplication signs from time to time to make the distinction between terms easier to see)

$$PTR = c \times ACS$$

The second basic relationship is

$$PTR = \frac{ATC}{I \times p_T}$$

This can be derived as follows:

Let the expenditure on teaching staff be £$_T$

The average teacher cost ATC is therefore given by

$$ATC = \frac{£_T}{T}$$

So

$$ATC \div PTR = \frac{£_T}{T} \times \frac{T}{N}$$

So

$$ATC \div PTR = \frac{£_T}{N}$$

But

$$\frac{£_T}{N} = I \times p_T$$

Where I is the income per pupil and p_T is the proportion of revenue expenditure on teaching staff

Hence

$$PTR = \frac{ATC}{I \times p_T}$$

The overall approach

1   Estimate the available revenue funding for the academic year in question
2   Calculate the current average teacher cost including on cost from actual salaries, on costs and FTE values. Use this to make a best estimate allowing for pay increases, pay progression, staff changes and any restructuring of the average teacher cost for the academic year in question. It may be useful to estimate a range, highest to lowest for this. This gives ATC for the equations above
3   Analyse all the non-teaching costs for the academic year in question and split them into essential and desirable elements to assess the overall range of flexibility between the upper and lower levels of expenditure on non-teaching costs. Use these results with the value for the available revenue to calculate values for p_T as defined for the equations above
4   Make a best estimate or best, most likely and worst estimates for the per pupil funding for the academic year in question from point 1) above and estimated roll numbers. This gives I for the equations above
5   Use

$$PTR = \frac{ATC}{I \times p_T}$$

To calculate the PTR that can be afforded

6   Use the teacher contact ratio to calculate the Average class size in the curriculum from $PTR = c \times ACS$

Plan the curriculum to match the value of ACS or estimate the teacher period budget from the first ACS equation or from the roll, PTR contact ratio and periods in the timetable week.

If a lower value of ACS is the only curriculum possibility work backwards to find the PTR required for that and hence the change in p_T or ATC needed to do that or if that is not possible the degree of overspend required shown as a change in I.

Staff deployment analysis

This is basically calculating cost envelopes for different areas of the curriculum in terms pf teacher periods by considering the number of groups the year or area will be divided into and for how many periods that will be. For instance in year 10 the school below has decided to use 16 periods of the week where the year is divided into 8 groups, probably a core of Maths, English, PE and possibly Science and then three 3 period option blocks with nine option choices in each block.

Affordable PTR	16
Pupil Roll	1000
FTE Teachers	62.5
Estimated Contact ratio	0.78
Periods in timetable cycle (week)	25
Affordable ACS	20.5
Teacher period (tp) budget	1219

Area	Roll	Tp	Notes
Year 7	145	137	5 groups for 21 periods, 8 groups for 4 periods
Year 8	155	158	6 groups for 21 periods, 8 groups for 4 periods
Year 9	150	158	6 groups for 21 periods, 8 groups for 4 periods
Year 10	170	209	8 groups for 16 periods 9 groups for 9 periods
Year 11	180	218	8 groups for 16 periods 10 groups for 9 periods
Year 12	105	150	35 subject groups at 4 periods plus 5 groups at 2 periods
Year 13	95	150	35 subject groups at 4 periods plus 5 groups at 2 periods
Learning Support (Ls)	n/a		
Intervention (Int)	n/a		
Other (e.g. flexibility)	n/a	39	Timetable flexibility for later use in Ls and Int
Totals	1000	1219	

With other detail

This table shows the Average Class Size for each area and also the number of FTE teachers employed to cover each area. This type of information is simple to set up in a spreadsheet for group discussion

Area	Roll	Tp	ACS	FTE
Year 7	145	137	26.46	7.03
Year 8	155	158	24.53	8.10
Year 9	150	158	23.73	8.10
Year 10	170	209	20.33	10.72
Year 11	180	218	20.64	11.18
Year 12	105	150	17.50	7.69
Year 13	95	150	15.83	7.69

Learning Support (Ls)	n/a			
Intervention (Int)	n/a			
Other (e.g. flexibility)	n/a	39	n/a	2.00
Totals	1000	1219	20.5	62.5

Note that the ACS in years 12 and 13 represents the average class size if all students attend all lessons. This is not likely and if students on average only attend 80% of lessons (i.e. have around 20% private study time) then the actual average size of teaching group is 14 in year 12 and 12.7 in year 13.

The FTE total for each year group can be calculated by dividing the tp allocation by the average teaching load. The average teaching load is the contact ratio multiplied by the periods in the timetable cycle, in this case $0.78 \times 25 = 19.5$

The numbers in the table are subject to rounding errors.

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